High Energy Theory Seminar @ Osaka University
Tuesday June 25 2019, Toyonaka, Japan

Non-perturbative conversion of Wilson coefficients from 4-flavor to 3-flavor theories using lattice QCD

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(RBC & UKQCD Collaborations)

Outline

- Introduction
  - RBC & UKQCD Collaborations and their researches
  - $K \to \pi\pi$ & direct CP violation
  - Direct CPV parameter Re(ε'/ε): SM vs EXP
  - Significant error sources in lattice calculation
  - NP matching of 3/4-flavor Wilson coefficients
- Spherical average of 2pt functions & operator renormalization
- NP conversion of 3/4-flavor Wilson coefficients

The RBC & UKQCD collaborations

BNL and BNL/RBRC

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Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
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Andrew Yong
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Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Projects in RBC/UKQCD

- SM test (to be compared with experiments)
  - Weak Decays:
    $K \to \pi\pi$, $K \to \pi ll$, $K \to \pi\nu\nu$,
    $\pi^0 \to e^+e^-$, $K_L \to \mu^+\mu^-$, $\pi^0 \to \gamma\gamma$,
    $B_s \to Kl\nu$, $B_s \to D_s l\nu$
  - Neutral meson mixings:
    K^0–$\overline{K}^0$, D_s–$\overline{D}_s$, B_s–$\overline{B}_s$
  - Hadronic contribution to muon g−2
    HVP, HLbL

✦ Weak Matrix Elements
✦ Decay constants
✦ Form factors
✦ CKM matrix elements
☆ Constraints on BSM

- Lattice code: GRID
- New algorithms

Neutral Kaon System

- Kaons
  - Charged: $|K^+\rangle = |\bar{s}u\rangle,\ |K^-\rangle = |\bar{u}s\rangle$
  - Neutral: $|K^0\rangle = |\bar{s}d\rangle,\ |\overline{K}^0\rangle = |\bar{d}s\rangle$ * strong eigenstates

  Not CP eigenstates: $CP|K^0\rangle = |\overline{K}^0\rangle,\ CP|\overline{K}^0\rangle = |K^0\rangle$

- CP eigenstates
  - $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right),\quad CP|K_1\rangle = +|K_1\rangle$: CP even
  - $|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right),\quad CP|K_2\rangle = -|K_2\rangle$: CP odd

- CP nature of decay modes
  - $|\pi\pi\rangle$: CP even
  - $|\pi\pi\pi\rangle$: CP odd

K_S & K_L in CP limit

- Large difference in lifetimes:
  - $m_K - 2m_\pi \approx 220$ MeV $\gg m_K - 3m_\pi \approx 80$ MeV
  - $K_S \equiv K_1$, $K_L \equiv K_2$ (in CP limit)
  - $\tau_S \approx 9 \times 10^{-11}$s, $\tau_L \approx 5 \times 10^{-8}$s
- Possible processes:
  - $K_S \to \pi\pi$ (CP even to CP even)
  - $K_L \to \pi\pi\pi$ (CP odd to CP odd)

CP-violation in K → ππ

- $K_L \to \pi\pi$ discovered (1964) 
- Scenarios of CP-violation

CP odd / CP even

$$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$$

$|K_2\rangle \to |\pi\pi\rangle$: ε' direct CPV

$|K_1\rangle \to |\pi\pi\rangle$: ε indirect CPV

$|\pi\pi\rangle$ CP even

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \epsilon - 2\epsilon' \qquad \eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \epsilon + \epsilon'$$

direct CPV

- discovered in 1993
- NA48 (CERN), KTeV (FNAL): $\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right) = 16.6(2.3)\times 10^{-4}$

SM prediction

- SM vs Exp.

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} = 1.38(5.15)(4.59)\times 10^{-4}$$

RBC & UKQCD, 2015

↓ Kitahara et al, 2016 Improvement of RG

$1.06(5.07)\times 10^{-4}$

Exp: 16.6(2.3) x 10^{-4} ⟷ 2.8σ

- 27% systematic uncertainties from various sources in A_0

$$A_I = \langle(\pi\pi)_I|H_W|K\rangle$$

$$= \frac{G_F}{2}V_{us}^*V_{ud}\sum_i \underbrace{[z_i(\mu)+\tau y_i(\mu)]}_{\text{pQCD}}\underbrace{Z_{ij}(\mu,a^{-1})}_{\text{LQCD (+pQCD)}}\underbrace{\langle(\pi\pi)_I|Q_i^{lat}(a^{-1})|K\rangle}_{\text{LQCD}}$$

SM prediction

- SM vs Exp.

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$$= \frac{G_F}{2}V^*_{us}V_{ud}\sum_i [z_i(\mu) + \tau y_i(\mu)] Z_{ij}(\mu, a^{-1}) \langle(\pi\pi)_I|Q_i^{lat}(a^{-1})|K\rangle$$

pQCD — LQCD (+pQCD) — LQCD

Operator renormalization (15%)
Window problem: $\Lambda_{QCD} \ll \mu \ll a^{-1}$
Solved by "step scaling"

3f/4f matching (12%)
⭐ Target of this talk

Other sources
✦ Discretization (12%)
✦ Finite Volume (13%)

N_f in Weak Hamiltonian

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu)$$

$$= \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

$$= \ldots$$

We can use either 3f or 4f for WMEs

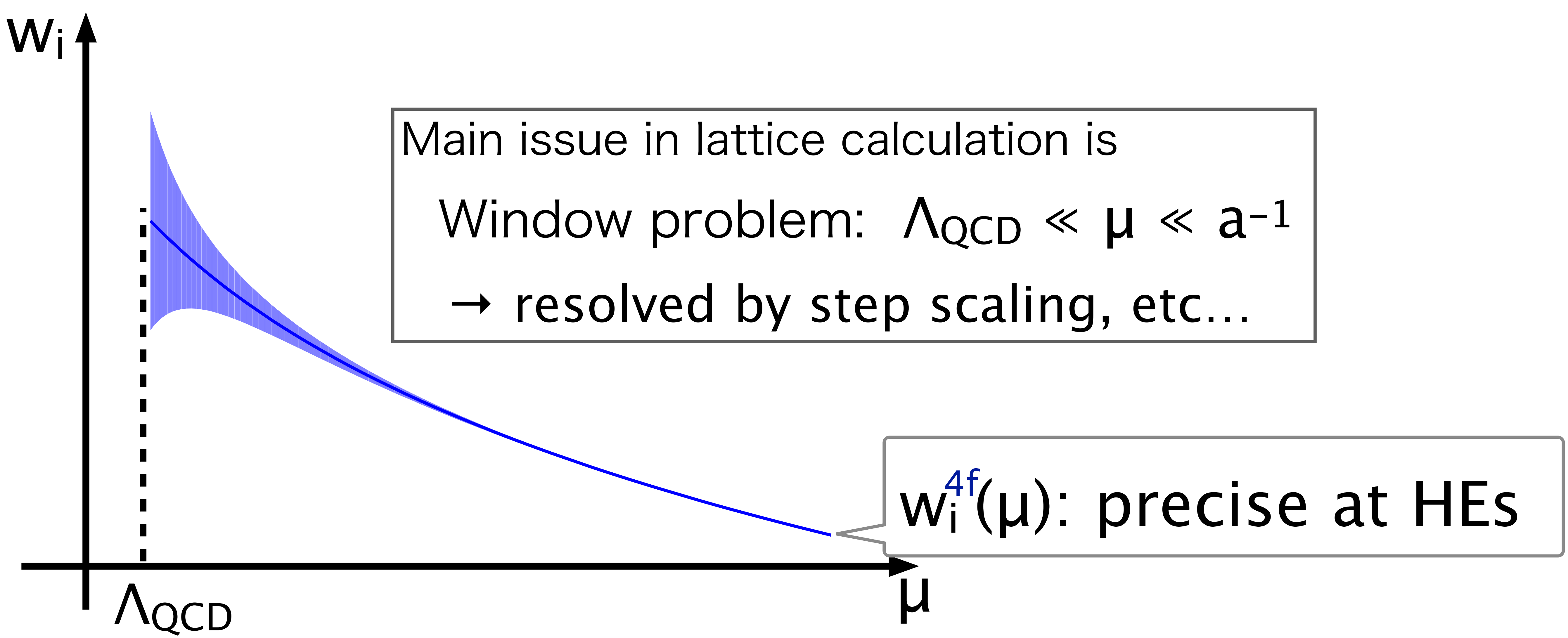
WMEs w/ 4-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{4f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{4f}(\mu)|i\rangle}_{\text{LQCD}}$$

Main issue in lattice calculation is

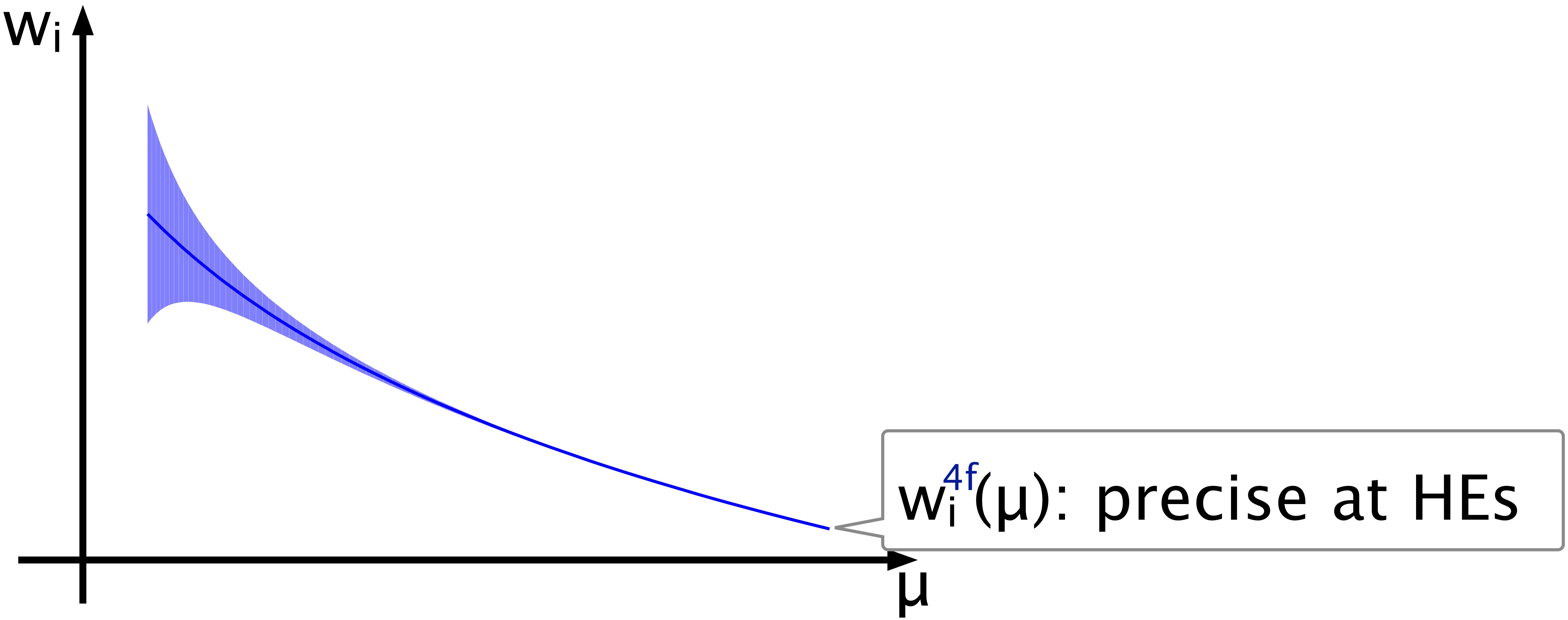
Window problem: $\Lambda_{QCD} \ll \mu \ll a^{-1}$

→ resolved by step scaling, etc…



WMEs w/ 3-flavor operators

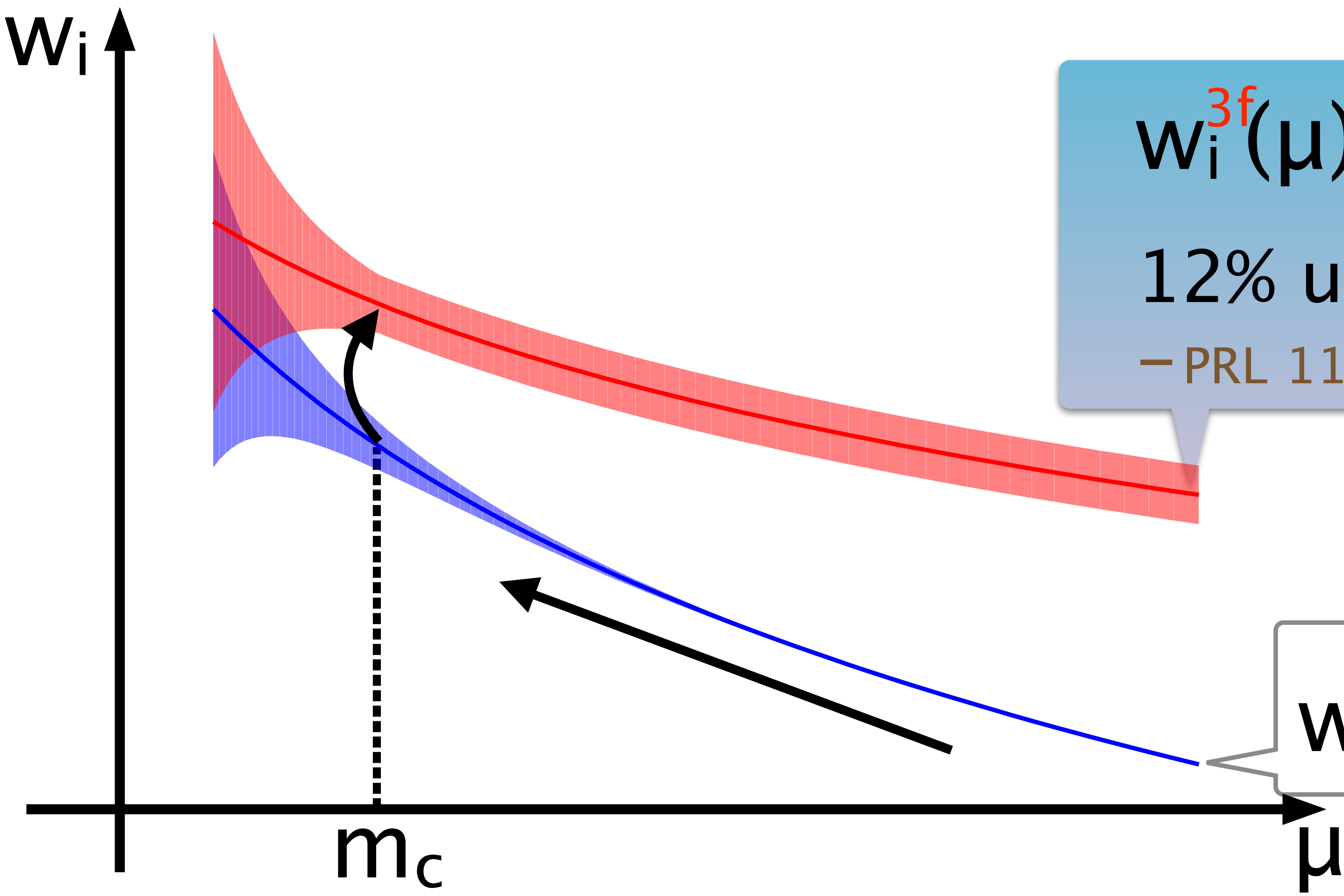
$$\langle f|H_W|i\rangle = \Sigma_i \, \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \, \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



WMEs w/ 3-flavor operators

$$<f|H_W|i> = \Sigma_i\, w_i^{3f}(\mu)\, <f|O_i^{3f}(\mu)|i>$$

pQCD LQCD



$w_i^{3f}(\mu)$: large uncertainty

12% uncertainty for $K \to \pi\pi$

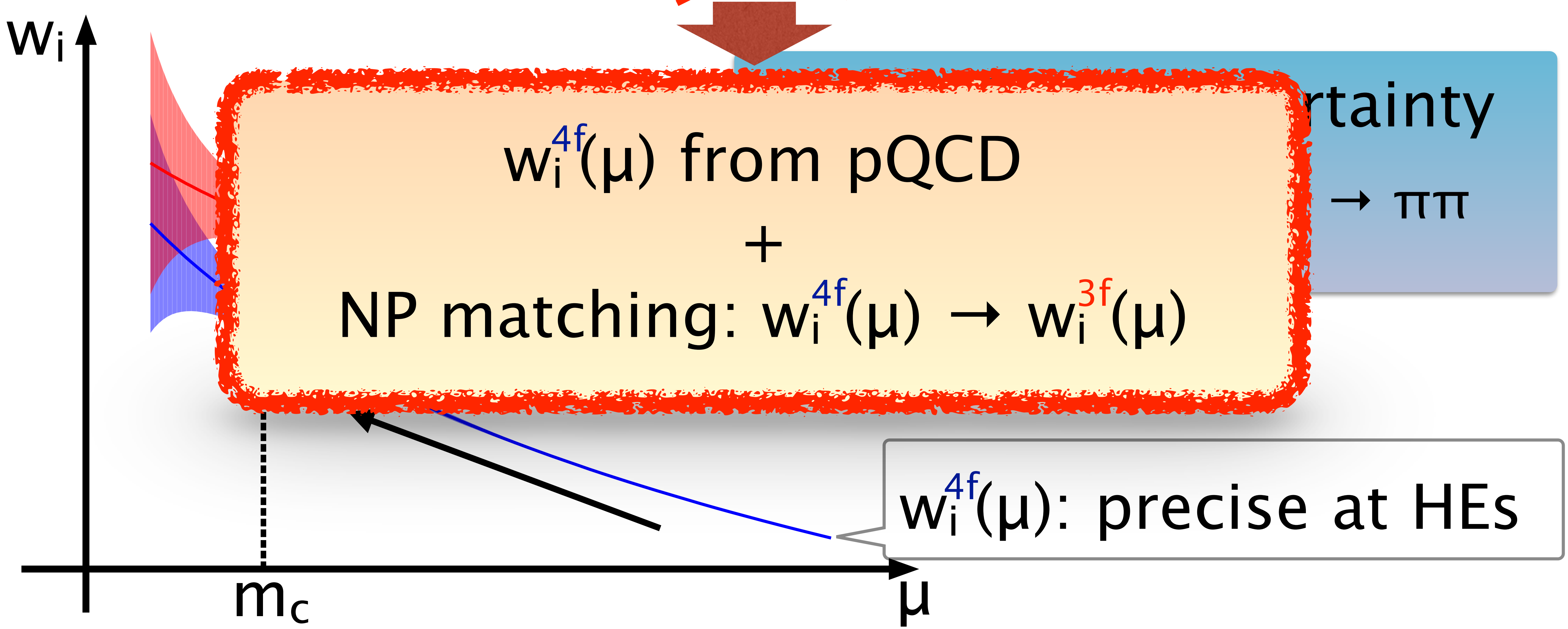
– PRL 115,212001(2015)

$w_i^{4f}(\mu)$: precise at HEs

WMEs w/ 3-flavor operators

$$\langle f|H_W|i\rangle = \Sigma_i\, w_i^{3f}(\mu)\, \langle f|O_i^{3f}(\mu)|i\rangle$$

~~pQCD~~ LQCD



$w_i^{4f}(\mu)$ from pQCD

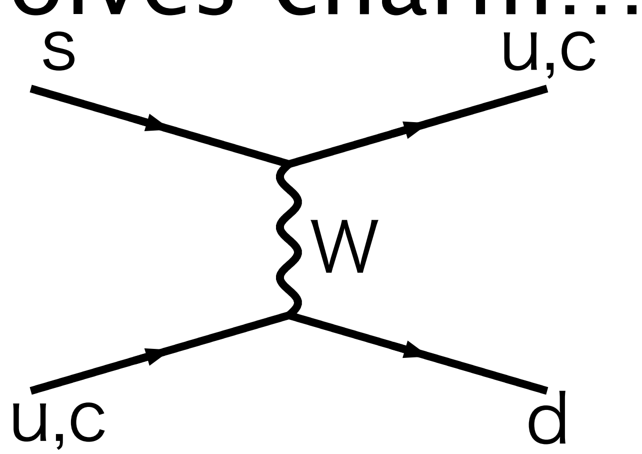
+

NP matching: $w_i^{4f}(\mu) \rightarrow w_i^{3f}(\mu)$

$w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$?

- Of coarse sea charm effects ⇒ $w_i^{3f}(\mu) \neq w_i^{4f}(\mu)$
  - Maybe small difference → neglect in this project
- If O_i^{4f} involves charm…

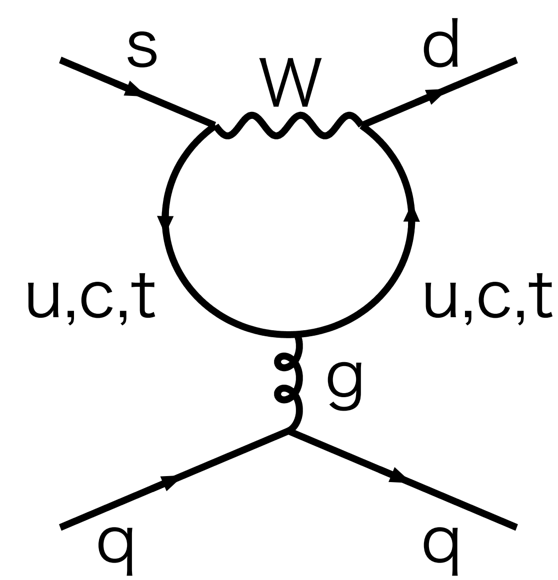
Ex)



current-current

$O_i^u = (\bar{s}d)_{V-A}\,(\bar{u}u)_{V-A}$

$O_i^c = (\bar{s}d)_{V-A}\,(\bar{c}c)_{V-A}$



QCD penguin

$O_i = (\bar{s}d)_{V-A}\,\Sigma_q\,(\bar{q}q)_{V\pm A}$

- Corresponding w_i's in 3f & 4f different
- w_i^{3f} necessary if MEs calculated with O_i^{3f}

$K \to \pi\pi$ by RBC/UKQCD (2015)

- 2+1 DWF
- a^{-1} = 1.38 GeV

  ⇒ too coarse to introduce charm

  ⇒ 3-flavor operators for MEs
  & perturbative 4/3-flavor matching

  ⇒ 12% systematic uncertainty in A_0

- NP matching (obtained from finer lattices) is desired

Outline

- [x] Introduction
- [ ] Spherical average of 2pt functions & operator renormalization
  - Euclidean correlators
  - Application to position-space renormalization
  - Construction of O(4)-symmetric 2pt functions
  - Continuum limit of renormalized mass
- [ ] NP conversion of 3/4-flavor Wilson coefficients

Euclidean correlators

$$\langle 0|O(x)O(y)^\dagger|0\rangle$$

- Good tool to extract information on O in QCD vacuum
  - $\sum_{\vec{x}}$: projection to $\vec{p} = 0$ state $\Rightarrow$ mass spectrum $\sim e^{-Mt}$
  - $E(\vec{p})$
- 3pt, 4pt functions also useful
  - scattering amplitudes
  - weak matrix elements
  - form factors
  - ...

Path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}U\,\mathcal{O}\ \mathrm{e}^{-S} = \frac{1}{Z} \int \mathcal{D}U \det D\ \mathrm{e}^{-S_G}\,\overline{\mathcal{O}}$$

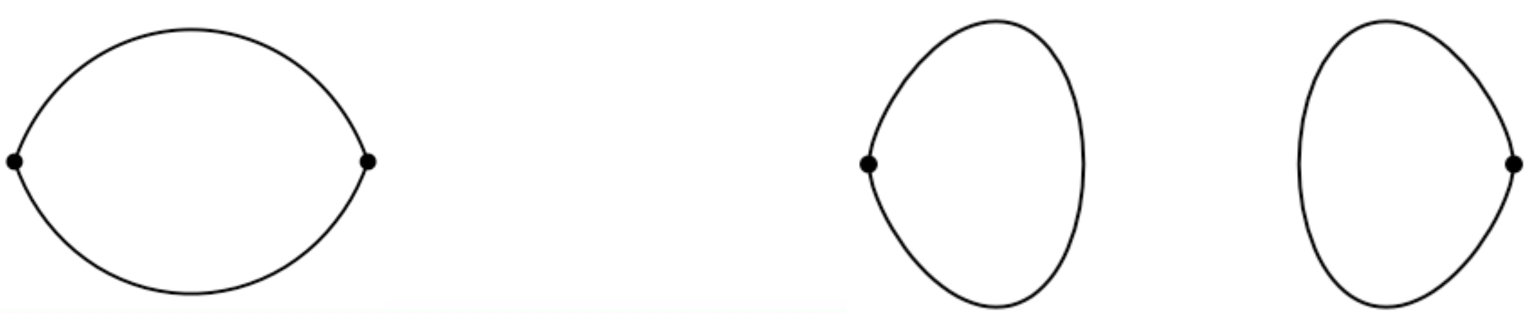
$$Z = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}U\ \mathrm{e}^{-S} = \int \mathcal{D}U \det D\ \mathrm{e}^{-S_G}$$

- $\mathcal{O} = \mathcal{O}(\overline{\psi}, \psi, U)$: Operator of interest
- $\overline{\mathcal{O}} = \overline{\mathcal{O}}(D^{-1}(U), U)$: Wick-contracted form of $\mathcal{O}$

Ex: $\mathcal{O} = \overline{\psi}(n)\mathrm{i}\gamma_5\psi(n) \cdot \overline{\psi}(0)\mathrm{i}\gamma_5\psi(0)$

$$\overline{\mathcal{O}} = n_f \mathrm{Tr}[(D^{-1})_{n,0}\gamma_5(D^{-1})_{0,n}\gamma_5] - n_f^2 \mathrm{Tr}[(D^{-1})_{n,n}\gamma_5] \cdot \mathrm{Tr}[(D^{-1})_{0,0}\gamma_5]$$

2 diagrams:



Operator Renormalization

- Necessary step before continuum limit

$$\left.\begin{array}{l} X^{lat}(a_1) \to Z^{R/lat}(\mu; a_1) X^{lat}(a_1) \\ \vdots \\ X^{lat}(a_n) \to Z^{R/lat}(\mu; a_n) X^{lat}(a_n) \end{array}\right\} \xrightarrow{a \to 0} X^R(\mu)$$

- Example
  - Quark mass

$$m_q^{lat}(a) \to Z_m(\mu; a) m_q^{lat}(a)$$

  - Matrix Elements

$$\langle f | O_i^{lat}(a) | i \rangle \to Z_{ij}^{R/lat}(\mu; a) \langle f | O_j^{lat}(a) | i \rangle$$

Mass renormalization w/ 2pt func.

- Scalar current renormalization useful

$$Z_m = Z_S^{-1} (= Z_P^{-1} \text{ if chiral symmetry in lattice fermions})$$

- Renormalization using 2pt functions
  - Renormalization condition

$$\left(\tilde{Z}_\Gamma^{\overline{MS}/lat}(\mu; a; x)\right)^2 \Pi_\Gamma^{lat}(a; x) = \Pi_\Gamma^{\overline{MS}}(a; x)$$

$$\Rightarrow \boxed{\tilde{Z}_\Gamma^{\overline{MS}/lat}(\mu; a; x) = \sqrt{\frac{\Pi_\Gamma^{\overline{MS}}(a; x)}{\Pi_\Gamma^{lat}(a; x)}}}$$

$$\Pi_S(x) = \langle \bar{u}d(x)\,\bar{d}u(0) \rangle$$

$$\Pi_P(x) = \langle \bar{u}i\gamma_5 d(x)\,\bar{d}i\gamma_5 u(0) \rangle$$

  - Advantages
    - ✦ Renormalization in a gauge invariant manner
    - ✦ $\overline{MS}$ calculation available to $O(\alpha_s{}^4)$

Chetyrkin & Maier, 2011

Lattice calculation

- Ensembles
  - 2+1 Domain-wall fermions
  - 3 lattice spacings: 1.7−3.1 GeV
  - Pion masses: 300−420 MeV

- For each ensemble we analyze

$$\widetilde{m}_q^{\overline{\rm MS}}(\mu; x; a) = \underline{m_q^{\rm bare,phys}(a)} \sqrt{\frac{\frac{1}{2}\left(\Pi_S^{\rm lat}(1/a; x) + \Pi_P^{\rm lat}(1/a; x)\right)}{\Pi_S^{\overline{\rm MS}}(\mu; x)}}$$

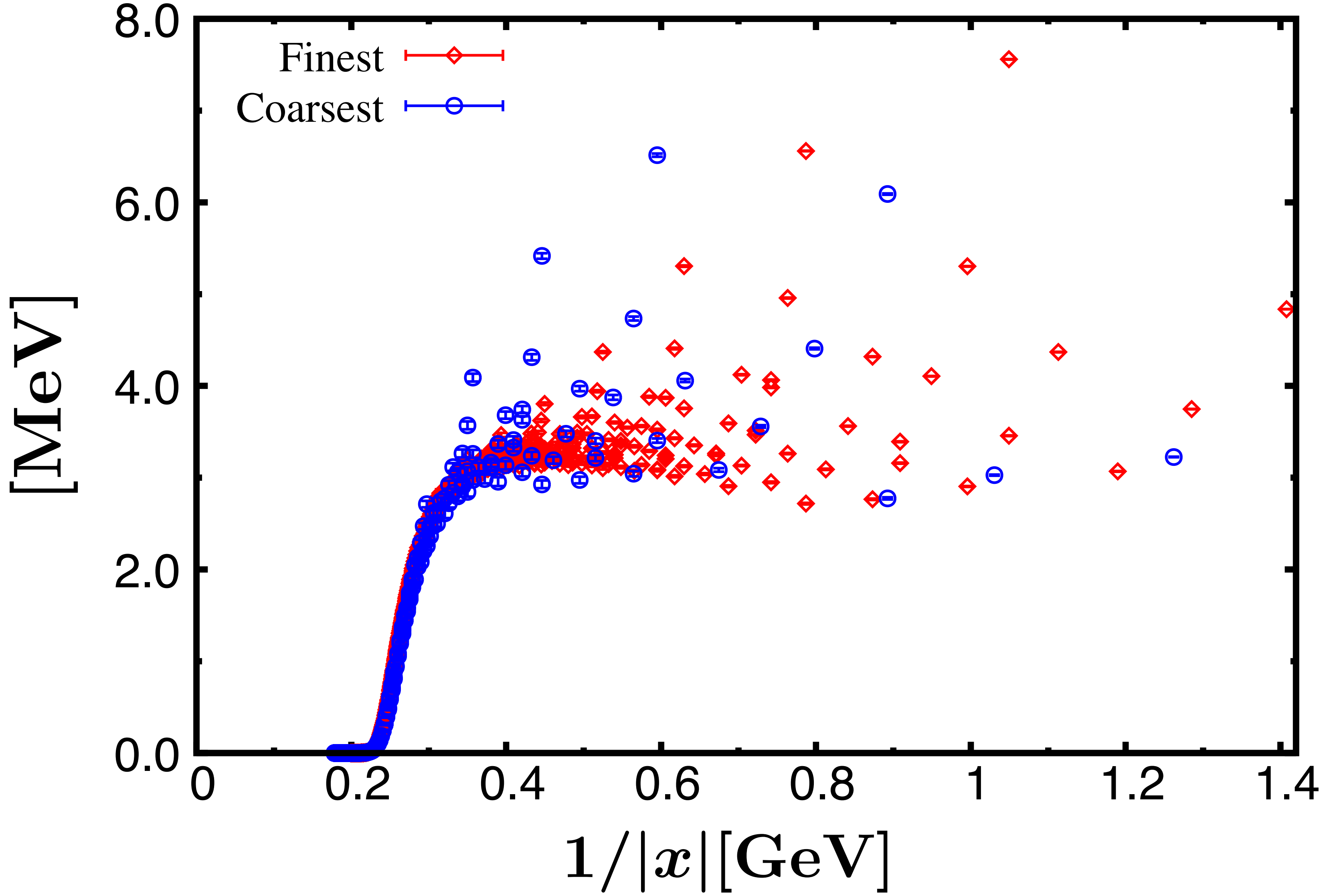
[RBC/UKQCD (2016)]

Z_m + irrelevant x-dependence

- Discretization effects (SDs)
- Non-perturbative effects (LDs)

→ extracted from intermediate region

$\tilde{m}_{ud}^{\overline{MS}}(3\,\mathrm{GeV};x)$



- Different lattice points distinguished ((1,1,1,1) vs (0,0,0,2))
- Large discretization errors
- How to take the continuum limit?

Average over spheres

MT & N.Christ, PRD99 014515

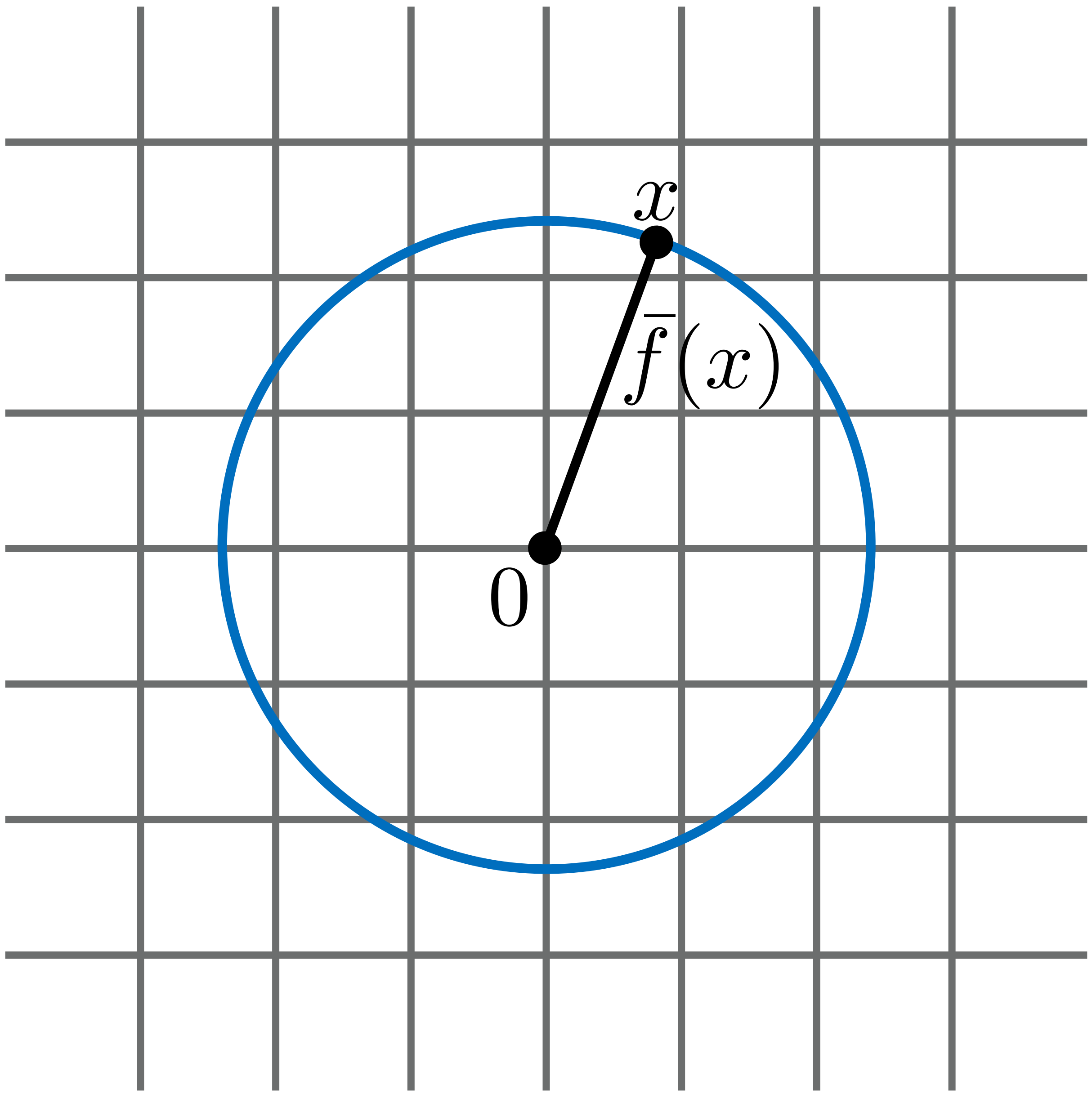
- Estimate the value of a quantity at each 4d point from values at lattice points, with a guideline

$$\bar{f}(x) = \eta(f^{\mathrm{lat}}; x)$$

※ details in following slides

- Take the average over the sphere for each distance |x|

$$\hat{f}(|x|) = \frac{1}{2\pi^2} \oint_{S^3(|x|)} \mathrm{d}\Omega\, \bar{f}(x)$$



Potential $O(a^1)$ error (1-dim)

- Defs:
  - f_n: lattice value at site n
  - $f(x)$: "continuum limit" : $f_n = f(an) + O(a^2)$

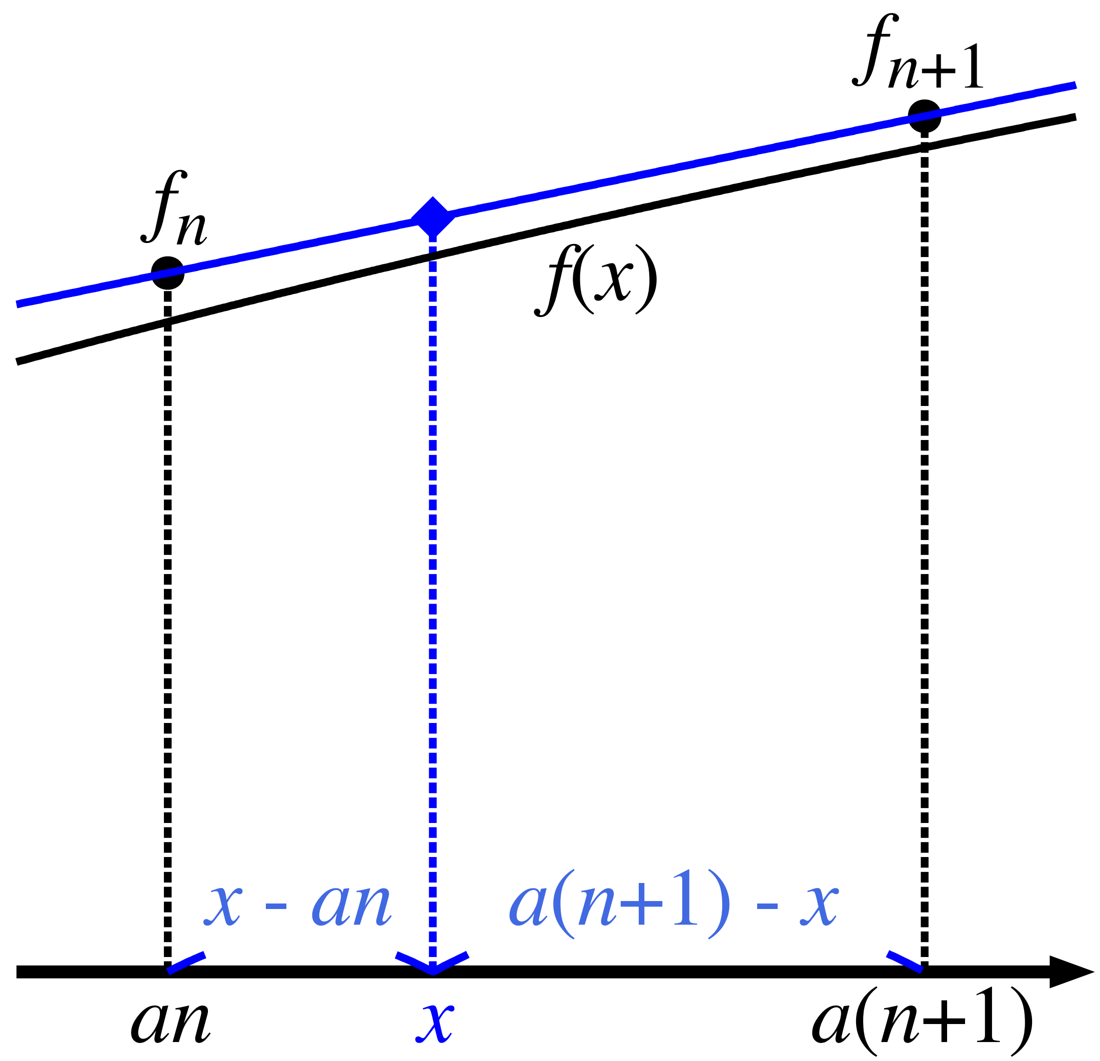
- Estimation $\bar{f}(x)$ should satisfy
  - $\bar{f}(x) = f(x) + O(a^2)$

- Potential $O(a^1)$ error in $\bar{f}(x)$
  - $f_n = f(an) + O(a^2)$

    $= f(x) + \underbrace{f'(x)\cdot(an-x)}_{O(a^1)} + O(a^2)$
  - $\bar{f}(x)$ is calculated using f_n's $\Rightarrow O(a^1)$ can appear
  - Balanced combination needed

Evaluation of $\bar{f}(x)$ (1-dim)

- Linear interpolation

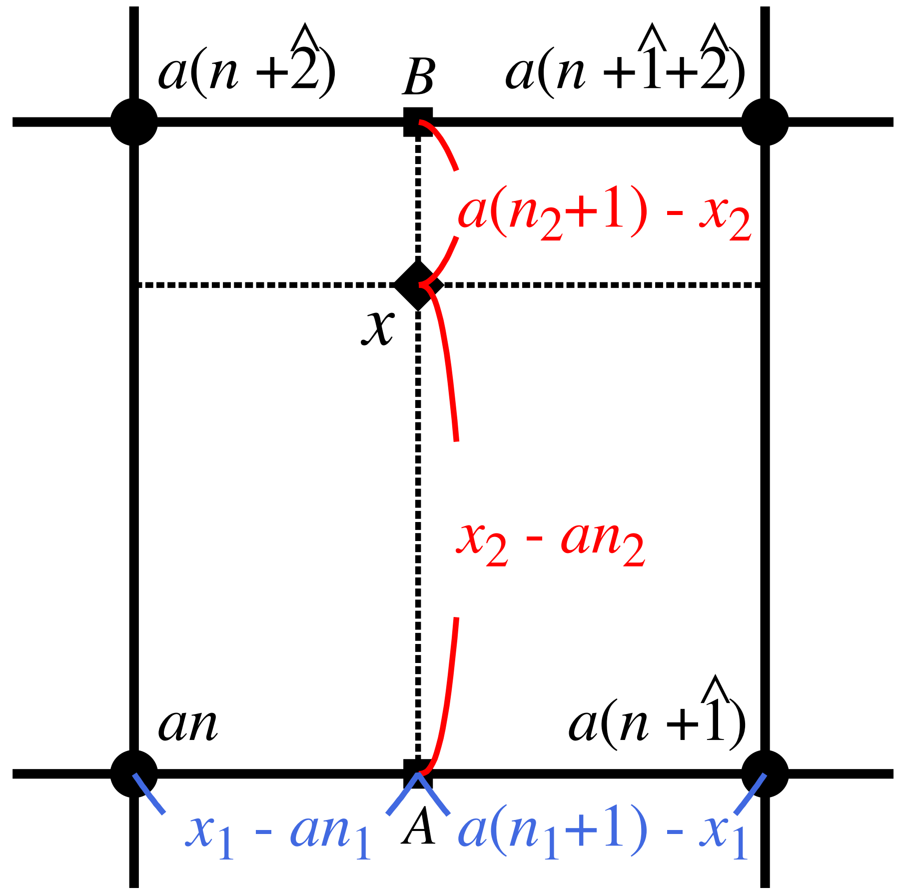


$$\bar{f}(x) = \frac{(a(n+1) - x)f_n + (x - an)f_{n+1}}{a} = f(x) + O(a^2)$$

Accurate up to O(a²)

Evaluation of $\bar{f}(x)$ (2-dim)

- Bilinear interpolation



$$\bar{f}(x) = \frac{(a(n_2+1)-x_2)\bar{f}(A) + (x_2 - an_2)\bar{f}(B)}{a}$$

$$= a^{-2}\begin{pmatrix} a(n_1+1)-x_1 & x_1 - an_1 \end{pmatrix}\begin{pmatrix} f_n & f_{n+\hat{2}} \\ f_{n+\hat{1}} & f_{n+\hat{1}+\hat{2}} \end{pmatrix}\begin{pmatrix} a(n_2+1)-x_2 \\ x_2 - an_2 \end{pmatrix}$$

$$= f(x) + O(a^2)$$

Evaluation of $\bar{f}(x)$ (4-dim)

- Quadrilinear interpolation

$$\bar{f}(x) = a^{-4} \sum_{i,j,k,l=0}^{1} \Delta_{1,i}\Delta_{2,j}\Delta_{3,k}\Delta_{4,l} f_{n+i\hat{1}+j\hat{2}+k\hat{3}+l\hat{4}}$$

$$\Delta_{\mu,i} = |a(n_\mu + 1 - i) - x_\mu|$$

– Accurate up to $O(a^2)$

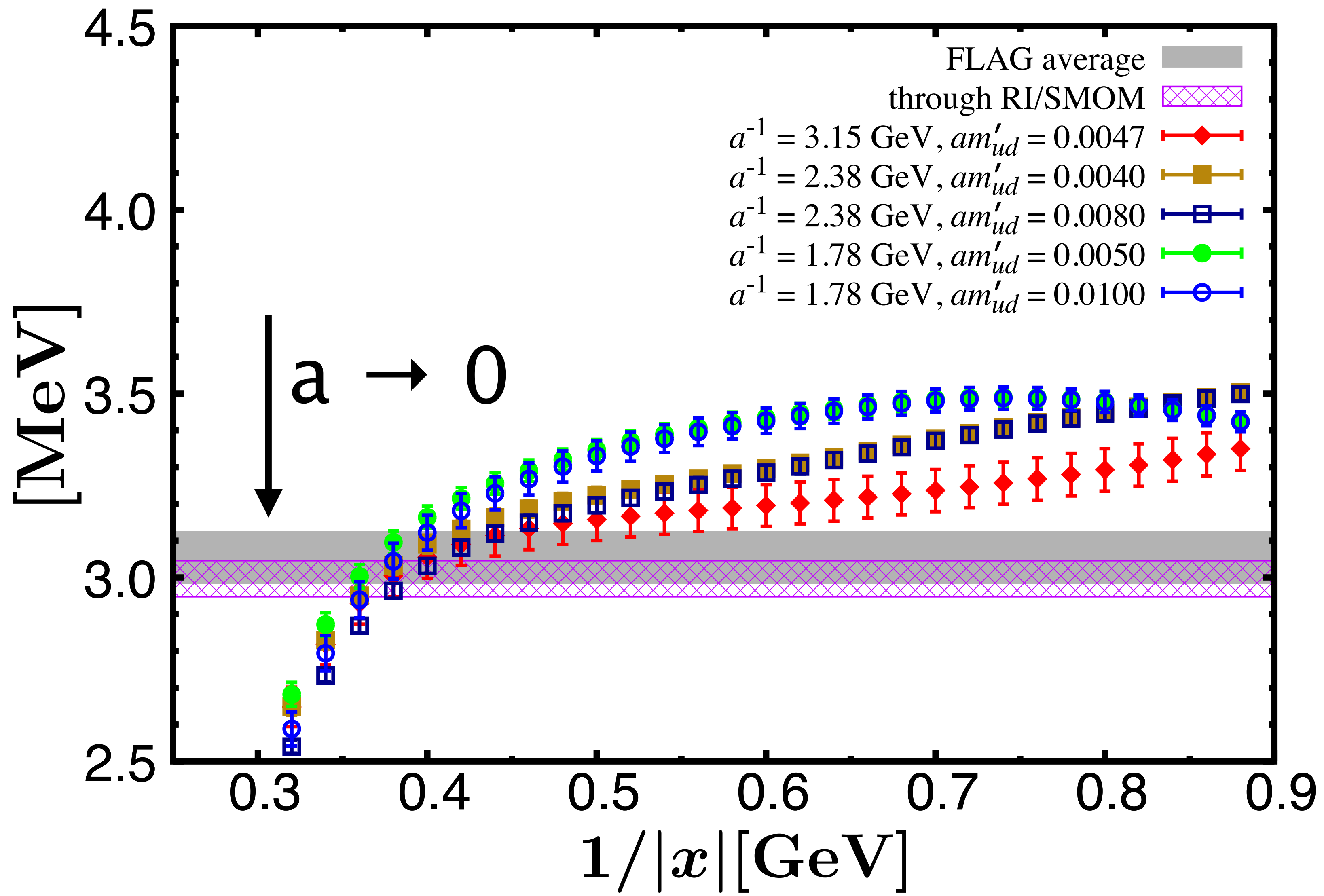
Result of spherical average

- Sphere average of

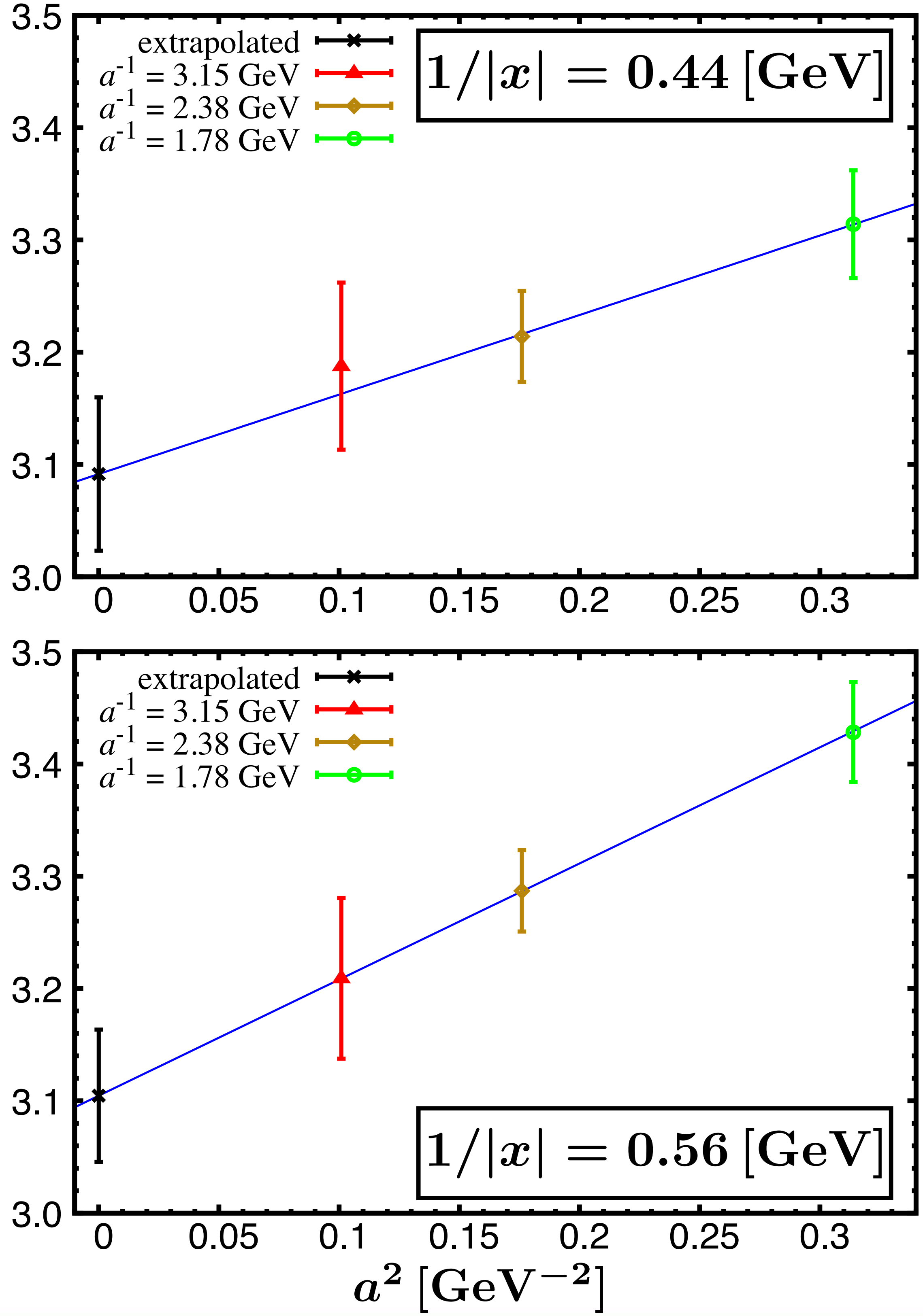
$$\widetilde{m}_q^{\overline{\mathrm{MS}}}(\mu;x;a) = m_q^{\mathrm{bare,phys}}(a)\sqrt{\frac{\frac{1}{2}(\Pi_S^{\mathrm{lat}}(1/a;x)+\Pi_P^{\mathrm{lat}}(1/a;x))}{\Pi_S^{\overline{\mathrm{MS}}}(\mu;x)}}$$

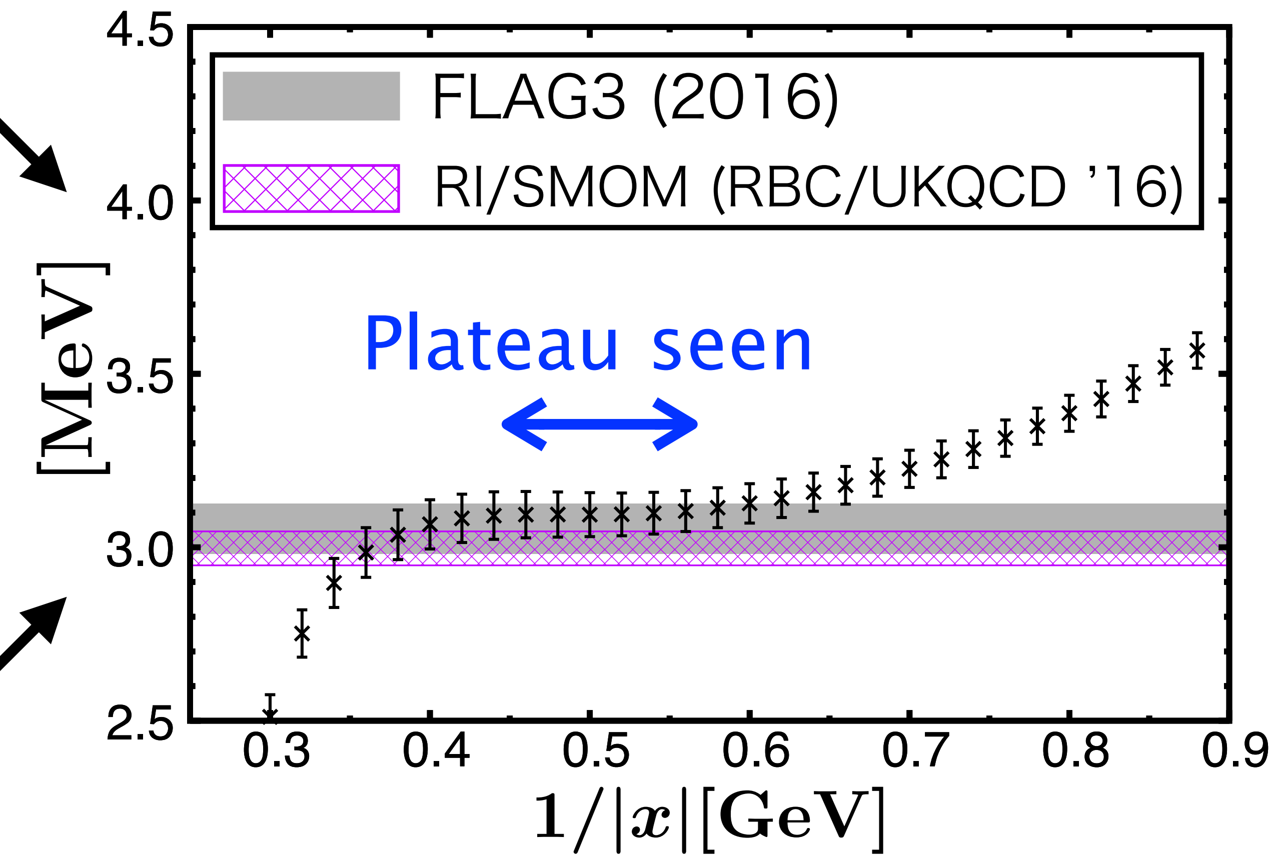
- Able to calculate at any distance

- Plateau seen better for finer lattices



Continuum limit





$$m_{ud}^{\overline{MS}}(3\ \text{GeV})|_{2+1f} = 3.09\ (6)_{stat}(6)_{sys}$$

$$m_{s}^{\overline{MS}}(3\ \text{GeV})|_{2+1f} = 85.3\ (1.6)_{stat}(1.7)_{sys}$$

Outline

- ☑ Introduction
- ☑ Spherical average of 2pt functions & operator renormalization
- ☐ NP conversion of 3/4-flavor Wilson coefficients
  - Gauge invariant procedure using 2pt functions
  - Result of an exploratory calculation

NP 4f-3f matching in position Sp.

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

- This means:

  $$\Sigma_i <\bar{O}(x)\, O_i^{4f}(\mu;y)^\dagger>\, w_i^{4f}(\mu) = \Sigma_i <\bar{O}(x)\, O_i^{3f}(\mu;y)^\dagger>\, w_i^{3f}(\mu)$$

  for any operator $\bar{O}(x)$

  at LDs: $1/|x-y| \ll m_c$

- Relation b/w w_i^{4f} & w_i^{3f} can be obtained by choosing appropriate number of $\bar{O}(x)$'s

  ⇒ We choose

  $$\bar{O}(x) = O_i^{3f}(\mu;x)$$

NP 4f-3f matching in position Sp.

$$H_W = \Sigma_i\, w_i^{4f}(\mu)\, O_i^{4f}(\mu) = \Sigma_i\, w_i^{3f}(\mu)\, O_i^{3f}(\mu)$$

$$\langle O_j^{3f}(\mu;x)\, H_W(y)^\dagger \rangle$$

$$\Sigma_j\, G_{ij}^{3f-4f}(\mu;x-y)\, w_j^{4f}(\mu) = \Sigma_j\, G_{ij}^{3f-3f}(\mu;x-y)\, w_j^{3f}(\mu)$$

$$G_{ij}^{nf-n'f}(\mu;x-y) = \langle O_i^{nf}(\mu;x)\, O_j^{n'f}(\mu;y)^\dagger \rangle$$

$$w_i^{3f}(\mu) = \Sigma_{jk}\, (G^{3f-3f}(\mu;x-y))_{ij}^{-1}\, G_{jk}^{3f-4f}(\mu;x-y)\, w_k^{4f}(\mu)$$

☆ Gauge invariant & free from contact terms

⇒ can prevent mixing with irrelevant operators

Matching Mtx & 3f operators

- $M_{ik} = \Sigma_j (G^{3f-3f}(\mu;x-y))^{-1}_{ij} G^{3f-4f}_{jk}(\mu;x-y)$
- Inverse matrix $(G^{3f-3f}(\mu;x-y))^{-1}_{ij}$ exists
  ONLY IF O_i^{3f}'s are independent with each other
  - Ex: ΔS = 1 weak operators not the case!

Type	Q_i
current-current	$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L$
	$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L$
QCD penguin	$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_L$
	$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_L$
	$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_R$
	$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_R$
EW penguin	$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\beta)_R$
	$Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\alpha)_R$
	$Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\beta)_L$
	$Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q (\bar{q}_\beta q_\alpha)_L$

Fierz symmetry
→ 3 relations among Q_i's
→ 7 independent operators

Matching matrix

- If we choose
  - $O_i^{3f} = (Q_1, Q_2, \ldots, Q_{n3})$
  - $O_i^{4f} = (Q_1, Q_2, \ldots, Q_{n3}, P_1, P_2, \ldots, P_{nc})$
  - P_i's contain charm / Q_i's don't

We now redefine Q_i's as independent operators

- Then $M = \left(G^{3f-3f}(x)\right)^{-1}\left(G^{3f-3f}(x) \;\vdots\; \langle Q(x)\, P(0)^\dagger \rangle\right)$

$$= \left(1_{n3\times n3} \;\vdots\; \overbrace{\begin{matrix} \bullet & \bullet & \cdots & \bullet \\ \bullet & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \bullet & \cdots & \cdots & \bullet \end{matrix}}^{nc\ (=4\ \text{for}\ K\to\pi\pi)} \right)$$

Represents how P_i's turn to Q_i's below charm threshold

ΔS = 1 4-quark operators

3-flavor

Type	Q_i
current-current	$Q_1 = (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta)_L$ $Q_2 = (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha)_L$
QCD penguin	$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_L$ $Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_L$ $Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} (\bar{q}_\beta q_\beta)_R$ $Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{3f} (\bar{q}_\beta q_\alpha)_R$
EW penguin	$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\beta)_R$ $Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\alpha)_R$ $Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\beta)_L$ $Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_q^{3f} e_q(\bar{q}_\beta q_\alpha)_L$

7 independent operators

4-flavor

Type	P_i
current-current	$P_1 = (\bar{s}_\alpha d_\alpha)_L(\bar{u}_\beta u_\beta)_L$ $P_1^c = (\bar{s}_\alpha d_\alpha)_L(\bar{c}_\beta c_\beta)_L$ $P_2 = (\bar{s}_\alpha d_\beta)_L(\bar{u}_\beta u_\alpha)_L$ $P_2^c = (\bar{s}_\alpha d_\beta)_L(\bar{c}_\beta c_\alpha)_L$
QCD penguin	$P_3 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_L$ $P_4 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_L$ $P_5 = (\bar{s}_\alpha d_\alpha)_L \sum_q^{4f} (\bar{q}_\beta q_\beta)_R$ $P_6 = (\bar{s}_\alpha d_\beta)_L \sum_q^{4f} (\bar{q}_\beta q_\alpha)_R$
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9 independent operators

Color trivialization by Fierz trf.

- Def: $(\bar{s}d)_L(\bar{q}q)_{R/L} = \bar{s}\gamma_\mu(1-\gamma_5)d \cdot \bar{q}\gamma_\mu(1\pm\gamma_5)q$

- Left-Left operators

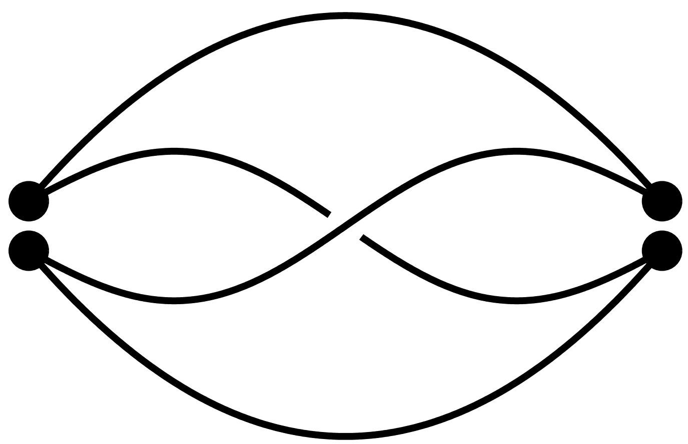
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_L = (\bar{s}_\alpha q_\alpha)_L(\bar{q}_\beta d_\beta)_L$$

- Left-Right operators

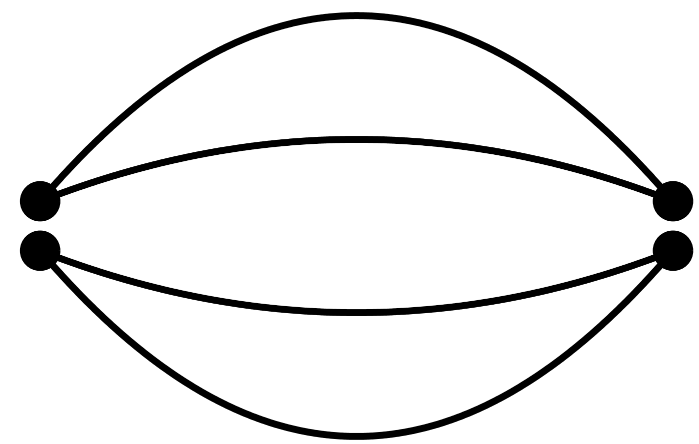
$$(\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_R = -2\bar{s}_\alpha(1+\gamma_5)q_\alpha \cdot \bar{q}_\beta(1-\gamma_5)d_\beta$$

Contractions

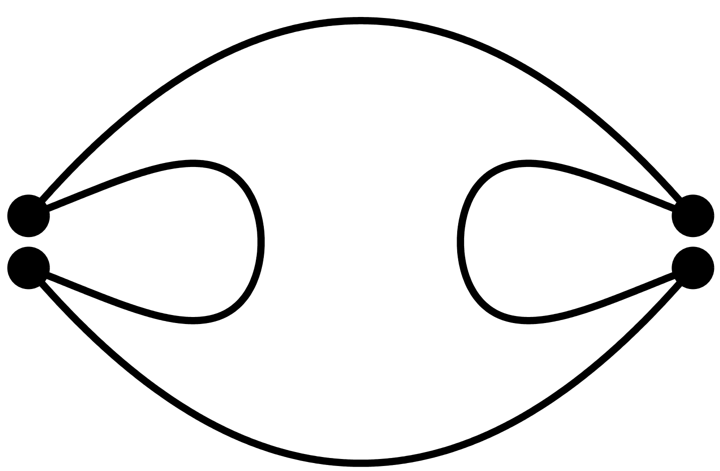
- 4/3-flavor matching should be independent of m_{ud} & m_s
  ⇒ Calculate w/ SU(3) valence quarks + 1 heavier quark



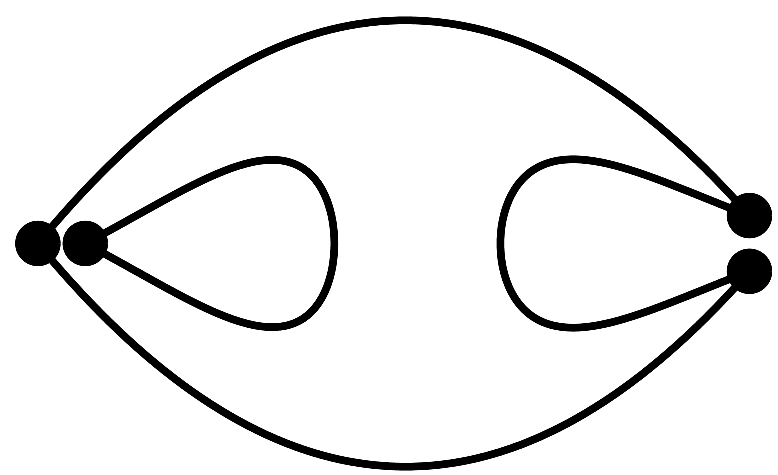
6 contractions



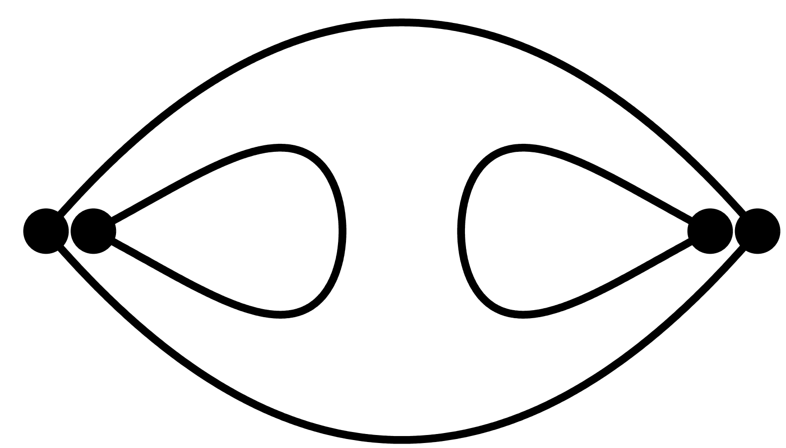
6 contractions



18 contractions



32 contractions



18 contractions

Subtraction of power divergence

- Loop diagram can contain power divergence
  - from power divergence of operators

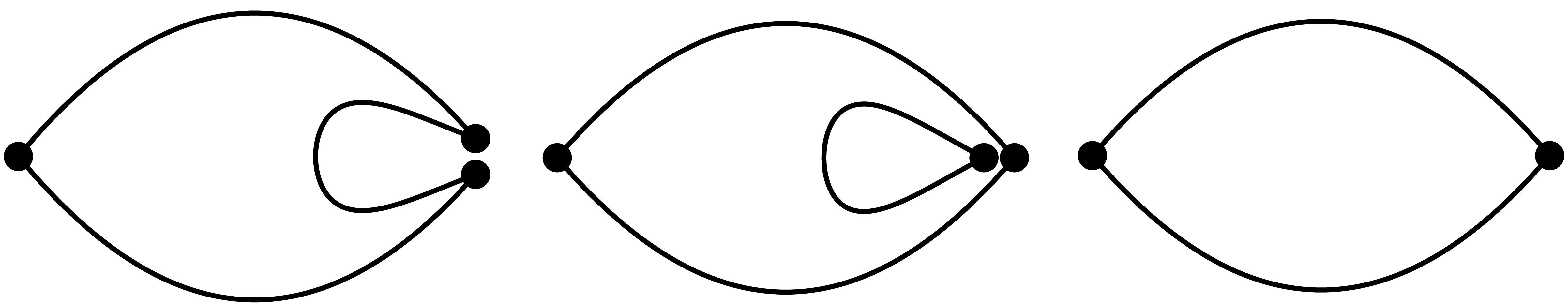
$$O_i \sim \frac{m_q}{a^2}$$

  - Eliminate by redefining

$$O_i' = O_i - C_- \bar{s}(1-\gamma_5)d - C_+ \bar{s}(1+\gamma_5)d$$

with a condition

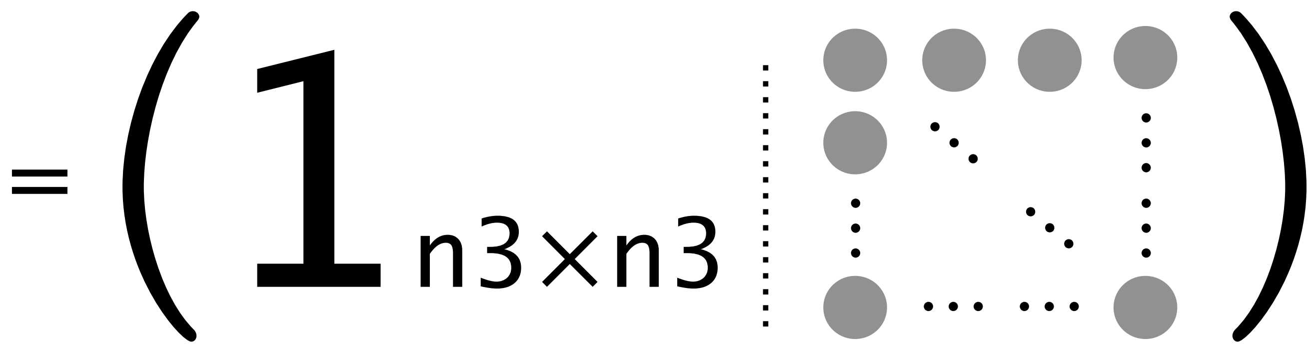
$$\left\langle \bar{s}(1 \pm \gamma_5)d(x) \cdot O_i'(y)^\dagger \right\rangle \Big|_{x-y=x_0} = 0$$



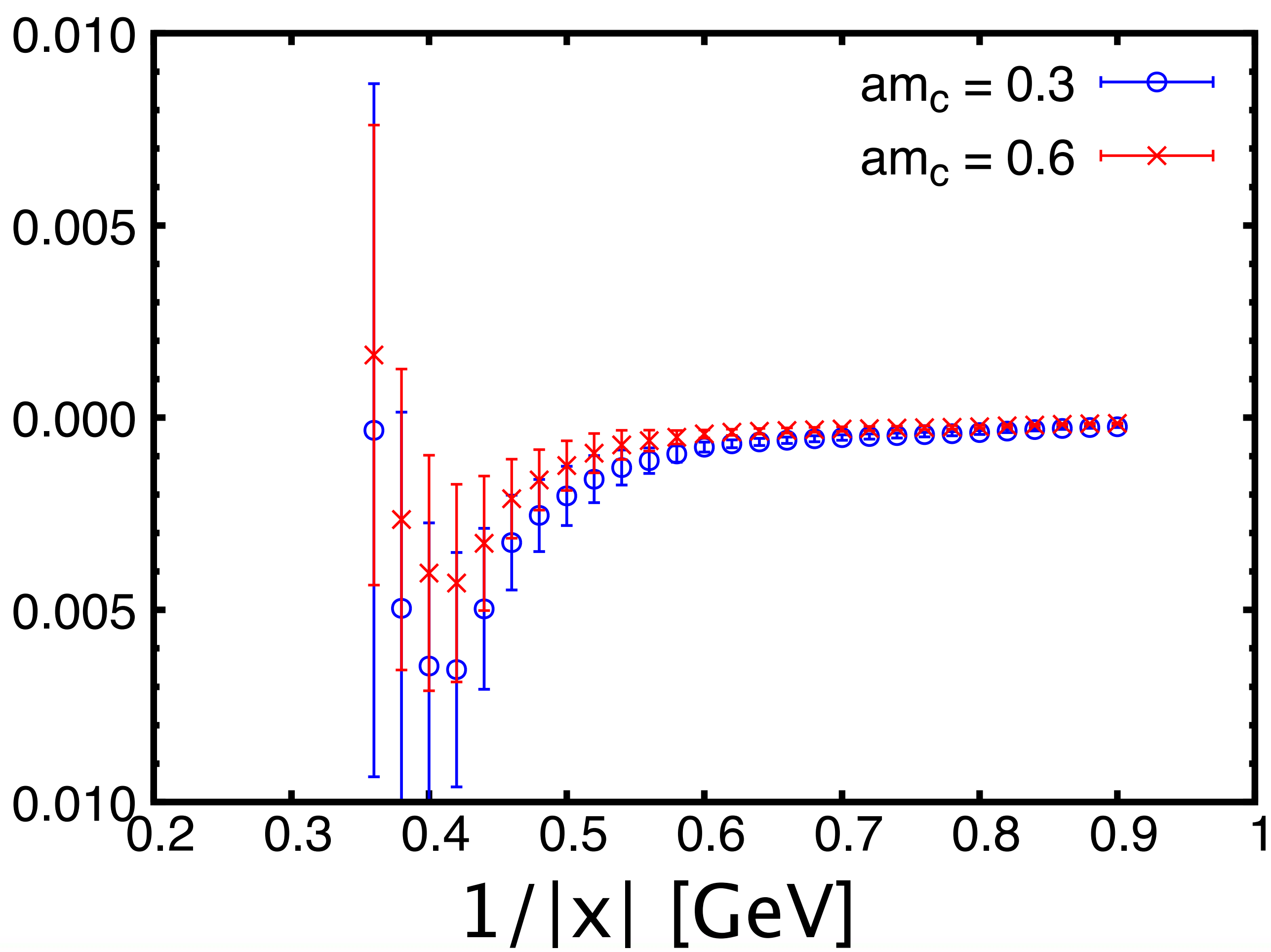
12 contractions 12 contractions 3 contractions

Result for M_{ij}

$$M_{ik} = \Sigma_j \, (G^{3f-3f}(x))^{-1}_{ij} \, G^{3f-4f}_{jk}(x)$$



– Valid and should be independent of x at LDs $|x| \gg 1/m_c$



- 16^3 x 32
- a^{-1} = 1.78 GeV
- 88 confs in 3,500 MD time
- $m_{ud}^{val} = m_s^{val} = m_s^{sea}$
- Unrenormalized

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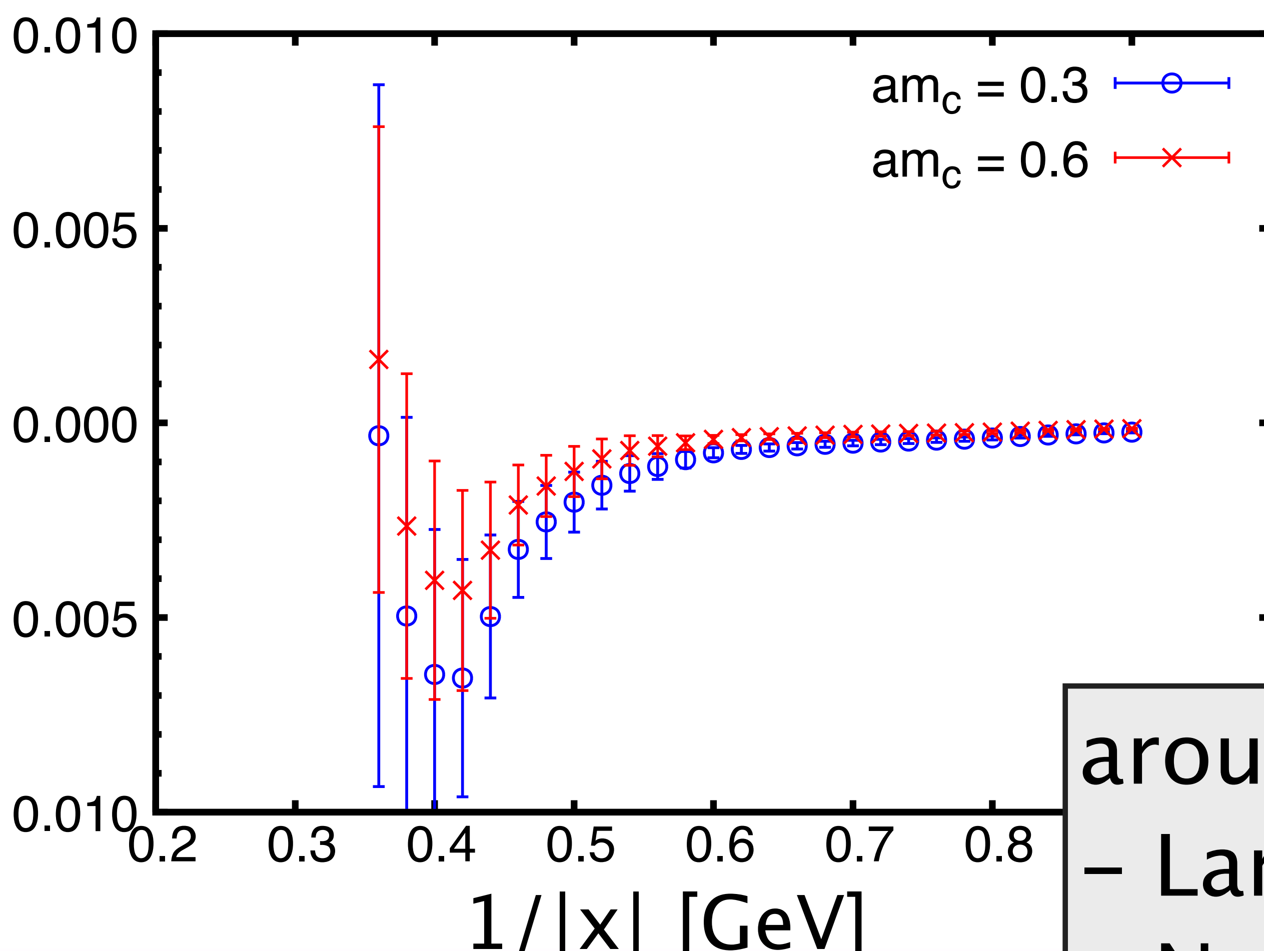

- $16^3 \times 32$
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- 88 confs in 3,500 MD time
- $m_{ud}^{val} = m_s^{val} = m_s^{sea}$
- Unrenormalized

around 1/|x| = 0.4 GeV…
– Large statistical error
– No clear plateau

Summary

- NP 4f/3f matching desired for $K \to \pi\pi$ calculation
- Position-space procedure
  - Gauge invariant
  - Free from mixing w/ irrelevant operators
  - Spherical average of 2pt functions
    - successful for quark mass renormalization
- Exploratory calculation on 16^3x32 lattice
  - Large statistical error
- To do
  - Seek ways to reduce statistical error (Lanczos A2A, …)
  - Main calculation on finer lattices (2.35 GeV, 3.15 GeV,…)

Previous effort in mom Sp.

- Condition

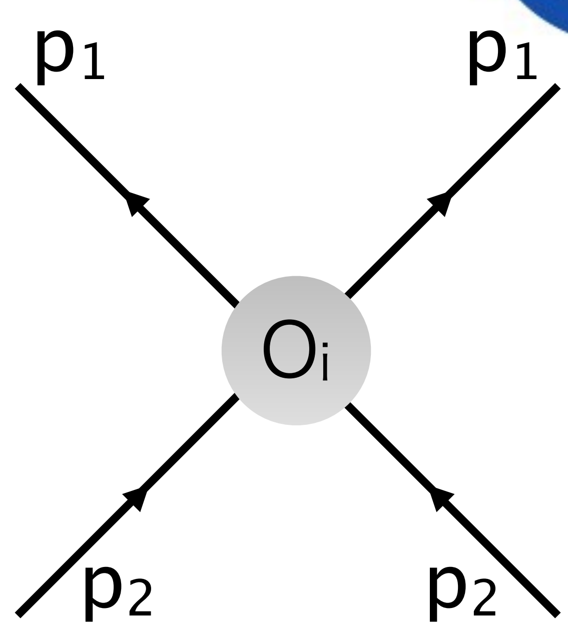
$$P^{abcd}_{\alpha\beta\gamma\delta}\ \Lambda^{abcd}_{\alpha\beta\gamma\delta}(O^{3f}_i(\mu);p_1,p_2)\ w^{3f}_i(\mu)$$

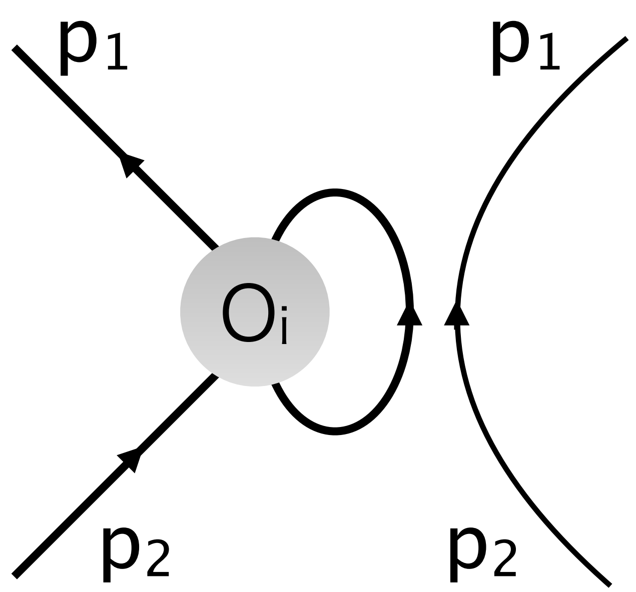
$$||$$

$$P^{abcd}_{\alpha\beta\gamma\delta}\ \Lambda^{abcd}_{\alpha\beta\gamma\delta}(O^{4f}_i(\mu);p_1,p_2)\ w^{4f}_i(\mu)$$

G-fixed amputated Green's function

flavor, color and spin projector





- Condition valid in $|p_{1,2}| \ll m_c$
- Statistical error
  - $|p_{1,2}| = 1.2$ GeV → 10%
  - $|p_{1,2}| = 0.6$ GeV → 50%
-

Why mom procedure so bad?

- Gauge fixing
  - Large Gribov noise
    - Gauge condition does not have a unique solution on the gauge orbit
    - Gauge-dependent quantities have some ambiguity
  - Mixing with gauge-noninvariant operators
- Off-shell condition
  - Mixing with operators that vanish by EoM

☆ All significant at small $p_{1,2}$

☆ Position-space procedure is free from all of these

Spherical average of a correlator